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NOTE ON THE ALGEBRAIC FORM $\frac{0}{0}$.

BY WM. CAIN.

When a fraction $\frac{F(x)}{f(x)}$ assumes the form $\frac{0}{0}$ for $x = a$, it is said to be indeterminate; but if it is assumed, for values of x differing as little from a as we please, that the fraction is determinate and can be computed, it is pertinent to enquire why the fraction assumes the form $\frac{0}{0}$ at $x = a$ and what is the meaning of it.

Chrystal in Algebra, Part II., page 66, gives the illustrative example, $\frac{x^2 - 1}{x - 1}$, which he says has no determinate value for $x = 1$ from any principles hitherto deduced. So the value of the fraction, for $x - 1$, is *defined* as the limit to which its value approaches as x approaches 1 indefinitely. Since we can divide numerator and denominator by $(x - 1)$ for any value of x other than 1, then for x not equal to 1,

$$\frac{x^2 - 1}{x - 1} = x + 1$$

which approaches 2 indefinitely as x approaches 1; so that by the definition, 2 is said to be its value when $x = 1$. This gives a value to the fraction for $x = 1$ which is continuous with the values assumed by it as x passes through the value 1, so that the fraction has a definite value corresponding to every value of its argument.

Additional light is thrown on the subject by McMahon and Snyder in their Differential Calculus, page 116, where the locus,

$$y = \frac{x^2 - a^2}{x - a} \quad . \quad . \quad . \quad (1)$$

$$\text{or. } (x - a)(y - x - a) = 0 \quad . \quad . \quad . \quad (2)$$

is plotted, and it is found to consist of the straight line $x = a$, parallel to axis of y and the straight line $y = x + a$, cutting the previous line at $(x = a, y = 2a)$.

When x is not equal to a, $y = x + a$, from (1) or (2) and corresponds to the ordinates of the straight line $y = x + a$. This is a continuous function of x, even when x passes through the value a. At $x = a$, this line has but one value, $y = a + a = 2a$. The other part of the locus, $x = a$, corresponds to any value of y and thus it is that (1) assumes the singular form $\frac{0}{0}$.

The point $(x = a, y = 2a)$ is the only one lying on both straight lines at the same time. The corresponding value of $y(= 2a)$ is exactly what has hitherto been defined as the value of the fraction; for on striking out the factor $(x - a)$ in the right member of (1), which is legitimate, by the ordinary rules of Algebra, so long as x is not equal to a, we get $y = x + a$, which approaches $y = 2a$ as x approaches a.

The treatment of this simple case can be extended to more complicated ones. Thus given,

$$y = \frac{x^2 - 8x + 5}{x^2 - 7x + 12} = \frac{(x - 3)(x - 5)}{(x - 3)(x - 4)} \quad . \quad . \quad . \quad (3)$$

The value of y can be computed for any value of x other than 3. For $x = 3$ owing to the factor $(x-3)$, y takes the form $\frac{0}{0}$ and the valve of y *ts said to be*, what $y = \frac{x-5}{x-4}$ approaches as x tends towards 3 or $y = (3-5) \div (3-4) = 2$. From (3),

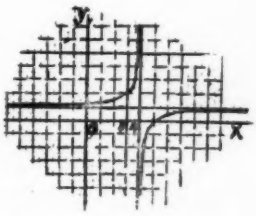
$$(x-3)[y(x-4) - (x-5)] = 0 \quad . \quad . \quad . \quad (4)$$

which is satisfied by the straight line,

$$x = 3 \quad . \quad . \quad . \quad (5)$$

and the hyperbola, $y(x-4) - (x-5) = 0$,

$$\text{or, } y = 1 - \frac{1}{x-4} \quad . \quad . \quad . \quad (6).$$



(5), or $x = 3$, is true for ny value of y, but (6) is continuaous in the neighborhood of $x = 3$, and when $x = 3$, y is exactly $1 - \frac{1}{x-4} = 2$. It is not a "limit" which the separate locus (6) approaches indefinitely and never reaches but a value that it actually attains and it is the intersection of the straight line (5) and the hyperbola (6).

It is this intersection, which is here a unique point $(x = 3,\ y = 2)$, whose ordinate is the value of y in (3) for $x = 3$, as hitherto *defined*.

Of course for $x = 3$, $y = \frac{0}{0}$ in (3) and can be anything, cor-

responding to points on the straight line (5) parallel to axis of y. And we see that to get the unique point ($x = 3$, $y = 2$) which is the intersection of both loci (5) and (6), we have simply to strike out the factor $(x - 3)$ in (3) and put $x = 3$ in the result. Thus we have the interpretation of the result as given by definition.

Generally, since $y = \dfrac{(x-a)\,F(x)}{(x-a)\,f(x)}$ takes the form $\dfrac{0}{0}$ for $x = a$, its value is defined to be the value of the right member when the factor $(x - a)$ is struck out and and x is put $= a$ in the result; *i. e.*, $y = \dfrac{F(a)}{f(a)}$ and it was so defined, because y approaches this value indefinitely as x approaches a.

The geometrical interpretation is found from the above equation in the form,

$$(x - a)[y(fx) - F(x)] = 0$$

to correspond to the intersection of the two loci:

$$x = a,\ y = \frac{Fx}{fx}.$$

The straight line $x = a$ may have any value of y, and the first equation above shows this by giving $y = \dfrac{0}{0}$ for $x = a$.

On striking out the factor $(x - a)$, the locus $y = \dfrac{Fx}{fx}$ is found, which assumes a *definite* value when $x = a$. It is one of the values of y that is actually attained. There is nothing illusory about it, provided $F(a) \div f(a)$ is real. It is assumed above that $(x - a)$ occurs only once as a factor in numerator and denominator. If it occur twice, the reasoning is as before and the condition unaltered. If it occur in $f(x)$ and not in $F(x)$, then $y = \dfrac{Fa}{fa} = \infty$ and the line $x = a$ is an

asymptote to the curve, $y = \frac{Fx}{fx}$.

A peculiarity here must be attended to. Thus if for equation (3) we substitute,

$$y = \frac{(x-4)(x-5)}{(x-4)(x-4)}$$

Then (5) is replaced by $x = 4$ and (6) by $y = 1 - \frac{1}{x-4}$, the same hyperbola as before.

Combining this equation with $x = 4$, we get $y = \infty$.

If x is first supposed less than 4, as it approaches 4, y is $+$ and increases indefinitely. For x slightly greater than 4, y is minus and increases numerically as x approaches 4. Therefore for $x = 4$, $y = +\infty$ or $-\infty$ and the function is discontinuous at $x = 4$.

If either $F(x)$ or $f(x)$ are irrational, either $F(a)$ or $f(a)$ may be imaginary, in which case the curve $y = \frac{Fx}{fx}$ does not intersect the line $x = a$, and the original fraction is imaginary for this value of x.

In any numerical example, it is easy to draw a figure and bring out the fact clearly that the fraction which takes the indeterminate form, for $x = a$, has an actual value, generally real when $x = a$, though this value is one of an infinite number which correspond to one of the loci $x = a$.

STUDIES OF BRAMBLES.

BY W. W. ASHE.

RUBUS CRUX. Stems few, or forming dense thickets 2-3 m. high, glabrous; at base erect, 2-4 cm. thick, obtusly 5-angled, sulcate between the angles, abundantly armed with very numerous, short, 1 cm. long, stout prickles; above sharply 5-angled, armed with very numerous, short 1 cm. long straight or upcurved prickles; tips spreading or reclining, armed with numerous shorter recurved prickles. Foliage glabrate or soft pubescent beneath; stem leaves 1.5-2.2 cm. long; stem leaves of 5 leaflets, terminal leaflet ovate or elliptic, cordate or rounded at base, abruptly acute, sharply serrate; lower pair of lateral leaflets sessile. Racemes loose, 8-12 cm. long; 8-14 flowered, the 2-3 cm.-long nearly glabrous pedicels spreading; flowers large, 3.5-4 cm. wide, the petals spatulate or oblong-ovate; fruit large, 4 cm. long, subglobose or ovate, black, sweet and juicy, ripening in July.

Dry hills, Buncombe County, N. C. Related to *R. argutus*, Link., from which separated by its larger size, larger racemes and glabrous stems. This plant and *R. immanis*. decribed below, are among the largest of the east American brambles.

RUBUS IMMANIS. Forming dense clumps 2-4 m. high. Stems stout, at first erect or ascending, then spreading or reclining, at least the tips; below terete or obtusely angled, 2-3 cm. thick, glabrous, densely armed with rather stout, 6-8 mm.-long, nearly straight, or recurved prickles; above terete, more slender, armed with similar prickles; the tips terete, glabrate or rough pubescent, armed with numerous short, stout, recurved prickles. Foliage thick, rough pubescent beneath, coarsely serrate, bright green, drooping; stem leaves 3 or 5-foliote, 1.5-2.5 cm. long; terminal leaflet broadly ovate or nearly orbicular, rounded or subcordate at base; lateral leaflets broadly ovate. Racemes glabrate or rough pubescent,

4-8-flowered, with one or rarely two leafy bracts; upper pedicels 2-3 cm. long, the lower longer, ascending. The flowers, nearly 4 cm. wide, appear the latter part of May. Fruit black, ovate or oblong, 3-4 cm. long, finely flavored, ripening in July and August.

Banks of the Watauga River, Watauga County, N. C., especially at and above Shulls Mill. Evidently related to *R. boyntoni*, proposed below, but far larger and lacking its long, prostrate tips.

RUBUS BOYNTONI. The base erect or ascending; above spreading or procumbent, the tips often trailing 7 m. to 10 m. Often forming dense thickets about 1 m. in height. Stems glabrous, villous or glandular pubescent, purplish or reddish; below terete, 1-1.5 cm. thick, armed with very numerous slender usually short prickles, or merely hispid, often mixed with stout gland-tipped hairs; the extensive trailing or reclining shoots, terete or angled, pubescent or glandular-pubescent, and armed with numerous weak prickles or bristles. Stem-leaves 3-foliate or sometimes 5-foliate, 1.5-2.5 cm. long; terminal leaflet 6-10 cm. long, 5-9 cm. wide, broadly oval or nearly orbicular, cordate at base; the lateral pair much smaller, sessile or nearly so; foliage dark green, dull, rather coarsely serrate, glabrate or somewhat pubescent. Inflorescence on short ascending branches, glabrate or pubescent, often glandular, racemose, 6-8-flowered; flowers large, white, 4-4.5 cm. wide. Fruit large, oblong-ovate, black when ripe in July, sweet and edible.

Dry soil in Haywood, McDowell, Buncombe and Madison Counties, N. C.

It gives me pleasure to associate with this plant the name of Mr. F. E. Boynton, of the Biltmore Herbarium, who many years ago pointed it out to me as worthy of study. While related to *R. trivialis* this bramble is well separated by its larger size, glandular pubescence and broader leaflets.

NEW NORTH AMERICAN THORNS.

BY W. W. ASHE.

TOMENTOSAE.

CRATAEGUS VERNANS. A slender tree 4-7 m. in height with dark nearly black scaly bark and purple-brown twigs freely armed with 3-4 cm-long thorns. Leaves ovate or nearly orbicular, 5-8 cm. long, abruptly acute at apex, rounded or cuneate at the entire base, sharply doubly serrate, few shallow ascending notches, thick, dark green above, glabrous or soft-pubescent beneath. Flowers, appearing the last of May in many-flowered usually glabrous corymbs, are about 15 mm. wide; stamens 20, nearly white. Fruit, borne in large compound clusters and ripening in October, in globose, 9-12 mm. thick, bright scarlet, capped by the acute deeply serrate lobes; seed 2-3, deeply grooved on inner face.

Morganton, West Virginia; Radford, Virginia. Separated from *C. Michiganensis*, the only other known member of this group with white anthers by having oblong leaves, and much smaller fruit.

CRATAEGUS OPICA. A slender tree, 4-7 m. in height, with short spreading branches forming an oblong or oval crown, the bark on the trunk nearly black, broken into small firm scales, or only cross-checked, that on the short branches lighter and smoother; the stem armed with many compound dark gray thorns: the stout glabrous red-brown twigs armed with 4-6 cm.-long stout purple-brown thorns. Leaves obovate or rhombic, 7-12 cm. long, 5-8 cm. wide, rounded or acute at the apex, narrow at the entire base, generally with 2-3 pairs of shallow notches, thick, firm, dark green and nearly glabrous above at maturity, remaining rough pubescent beneath, turning dull yellow, then blackened, and falling before the fruit the middle of October or earlier. Inflorescence a many-flowered more or less rough pubescent corymb; flowers small

14-16 mm. wide, calyx and the deeply glandular-serrate lobes pubescent; stamens 20, small, anthers pale yellow: styles 2 or 3. Fruit, borne in large compound clusters on pubescent nodding or drooping pedicels, is depressed-globose, thicker than long, about 12 mm. thick, glabrous, crimson, wax-like when fully ripe in October, and capped by the broad reflexed dark brown lobes; seed mostly 3, the inner faces with deep oblique grooves; flesh soft, pulpy, waxy, bright red.

From *Crataegus succulenta* Link the species above proposed is separated by having pale-yellow and not pink anthers; from *C. neo-fluvialis* Ashe by the shorter and comparatively broader foliage, and yellow, not dark purple anthers; from *C. Michiganensis* Ashe by its ovate or rhombic and not obovate foliage, and yellow, not white anthers. The type material is from Berks County, Pennsylvania, where collected by C. L. Gruber and W. W. Ashe.

CRATAEGUS FALLAX. A slender tree 7-8 m. in height with an oblong crown of short ascending branches, rough nearly black bark, glossy orange or light brown ascending twigs, at first tomentose, at length glabrous. Leaves broadly ovate or orbicular, 4.5-6 cm. long, finely serrate with sharp ascending teeth except at the broadly cuneate or rounded base, sometimes with a few broad shallow notches above the middle, abruptly acute, thick, at first pubescent below, soon glabrate except in the axils of the veins, dark green, turning dull yellow or black early in October. Inflorescence a many-flowered wide-spreading pubescent corymb; stamens 10. The fruit, borne in compound glabrous corymbs, on 4-6 cm.-long slender pedicels, ripening late in October and falling after the leaves, is depressed-globose, 13-16 mm. wide, swollen at the base, glossy crimson, mottled with green and olive, sparingly glaucous, capped by the broad entire ascending lobes; calyx tube very broad, slightly projecting; flesh greenish, hard; seed 2-3 furrowed on the broad rounded back, an oblique groove on the outer edge of the face.

Rocky hills, near Port Huron, Michigan; C. K. Dodge and

W. W. Ashe. From *C. integriloba*, Sarg. the above proposed species is separated by the smaller and less lobed leaf, and larger mottled green and late ripening fruit.

CRATAEGUS STRUCTILIS. A tree 5-6 m. in height with slender branches forming a broad oval crown, dark gray scaly bark on the armed trunk, and slender pubescent light brown twigs. Leaves oblong, ovate or obovate, 6-10 cm. long, 4-8 cm. wide, cuneate at the entire base, sharply doubly serrate and laciniately lobed above the middle, tomentose beneath, soon glabrate above, dark green; the short, 1-2 cm. long, stout petiole pubescent. The flowers, appearing the first week of June in large compound many, 15-30-flowered, tomentose corymbs, are 14-17 mm. wide; stamens 20, anthers light yellow; calyx lobes linear, serrulate, minutely glandular at base. The fruit, borne in compound, few-12-fruited, tomentose clusters, ripening late in September and falling with and soon after the leaves, is oblong or pyriform, 8-12 mm. long, 7-10 mm. thick, orange or reddish-orange, dotted with dark green and russet, the lobes usually deciduous, or, if persistent, reflexed; the narrow tube long-projecting; flesh soft, waxlike, orange; seed usually 2, apical, shallow-grooved on the round back.

Berks County, Pennsylvania; C. L. Gruber and W. W. Ashe. From *C. cerca*, which also has pyriform fruit and 20 stamens, *C. structilis* can be separated by its incisely lobed leaves and yellow, not purple anthers.

UNIFLORAE.

CRATAEGUS PENTANEURA. A shrub seldom more than 5 dm. in height with slender red-brown twigs, villose when young, at length scantily pubescent, armed with 2 cm. long ascending thorns. Leaves thin for the group, firm, dark green, glabrous or with a few scattered hairs on the pale lower surface especially along the veins, soon glabrous above, oval or obovate, 3-5 cm. long, 3-7, usually 5 prominent impressed veins, finely dentate and sometimes lobed. Flowers, ap-

pearing early in April in middle North Carolina and the last of May in New Jersey, solitary or 2 or 3 together on very short pubescent pedicels, are glabrate and about 14 mm. wide; lobes broadly lanceolate, finely glandular serrate, reflexed after anthesis. Fruit subglobose, 11-13 mm. thick, dark red, ripening early in October, the broad serrulate lobes reflexed; seed usually 5, a shallow groove down the narrow flat back, 5-6 mm. long.

Cabarrus County, North Carolina, W. W. Ashe; Hammonton, New Jersey, C. F. Saunders. Separated from the other known species by the nearly glabrous foliage and fruit and finely serrate lobes.

CRATAEGUS INDIGENS. A shrub seldom 1 m. in height with slender red-brown twigs at first gray tomentose. Leaves thick, dark green, coarsely dentate or doubly dentate above the middle, seldom lobed, the long cuneate base entire. The flowers, which appear in the vicinity of Chapel Hill, North Carolina, the last half of April, are about 14 mm. wide, and solitary, or 2, rarely 3 together on short tomentose pedicels; calyx lobes broadly lanceolate coarsely lobed, longer than the petals. Fruit, ripening in October, is globose, 12-14 mm. thick, orange, yellow or dull red, capped by the dull brown or yellow broad thin spreading or ascending lobes; seed usually 5, shallow-grooved on the narrow back.

Common on sandy soils at Chapel Hill, and Glendon, Moore County, North Carolina.

MOLLES.

CRATAEGUS VESCA. A tree attaining a height of 11 m. and a trunk diameter of 4 dm. with a broad oval crown of many spreading branches, dark brown furrowed bark on the unarmed trunk, and a slender olive or russet nearly glabrous twigs beset with few 2-3 cm.-long thorns. Leaves rather thin, firm, lucid, bright green, nearly orbicular, 6-9 cm. long, 6-8 cm. wide, rounded or obtuse at apex, obtuse or rounded at the entire base, crenate, with a few very short notches, soon becoming nearly glabrous beneath, except on the promi-

nent straight veins; petiole very slender 3-7 cm. long, pubescent or becoming glabrate. Flowers in large wide-spreading villose clusters; stamens small, 20. Fruit, borne in generally simple few-fruited clusters, ripening and falling early in October with the leaves, is subglobose, slightly thicker than long, 18-24 mm. thick, scarlet, nearly glabrous, capped by the narrow reflexed serrate pubescent lobes; flesh firm, white, sweet and juicy, very fragrant. Seed small and thin, usually 5.

C. vesca is one of the largest and most beautiful species of the genus and is conspicuous, even among other *molles*, on account of its glossy foliage and abundant brightly colored fruit. It grows on fertile slopes and bluffs along the Mississippi River, near Keokuk, in southeastern Iowa, where the type material was collected, and it will probably be found in the adjacent portions of Illinois and Missouri. It is at once separated from the other species by the small glossy glabrate nearly orbicular foliage.

CRATAEGUS REDOLENS. A tree 5-8 mm. in height with dark brown unarmed trunk, and stout red-brown twigs. Leaves broadly oval or nearly orbicular, 6-10 cm. long, 5-9 cm. wide, rounded or on vigorous shoots subcordate at the broad base, acute or obtuse at the apex, sharply doubly serrate, often with a few short broad spreading lobes, thick and leathery, dark green above, the upper surface at first rough pubescent, but soon glabrous, the lower surface petioles and young shoots at first densely tomentose, at length merely soft pubescent except on the veins and base of petiole. Inflorescence a large many-flowered compound tomentose corymb; flowers which appear about the middle of May, 22-24 mm. wide, stamens 20, anthers pale yellow; calyx with long attenuate tomentose lobes. Fruit, borne in large compound clusters, on long drooping pedicels, ripening late in September and falling with and after the leaves, is globose, 15-20 mm. thick, dark red, sparingly glaucous, barely pubescent at the ends; tube slightly projecting, the long narrow sharply serrate lobes ascending or spreading.

Iowa, Southern Minnesota and Wisconsin. L. H. Pammell, Clinton, Iowa; W. W. Ashe, Carlton, Minn.; W. W. Ashe, near Milwaukee, Wis. A late ripening species which resembles *C. altrix* Ashe in its fruit, and from which it can be separated by its thick broad seldom lobed foliage, and thicker more persistent tomentum, and larger ascending calyx lobes.

CRATAEGUS AMABILIS. A tree 6-8 mm. in height with dark furrowed unarmed bark on the trunk, stout villose russet or chestnut usually unarmed twigs, and very large globose or ovate brown or red-brown buds with large obtuse pubescent scales. Leaves ovate, 7-12 cm. long, 6-10 cm. wide, rounded or broadly cuneate at the base, seldom with a few irregular shallow notches, coarsely dentate, densely white villose on the lower surface; petiole stout, 2-4 cm. long, villose. Inflorescence a large compact villose corymb; lobes, deeply glandular serrate, erect after anthesis; stamens 20. Fruit, borne in small, mostly simple clusters, ripening and falling in September before the leaves, is globose, 20 mm. thick, bright scarlet or red, pubescent at the ends, the short broad lobes connivent over the broad deep cavity; flesh thick, firm, sweet, yellow; seed 5, narrow across the ungrooved back.

Bluffs of the Mississippi River at Keokuk, Iowa. In its oval villose leaf with few lobes and rounded base *C. amabilis* resembles *C. obtecta* Ashe, from which its stouter twigs, larger pubescent buds, and globose not oblong fruit separate it.

CRATAEGUS VALENS. A tree not more than 8 m. in height with firm rough brown bark on the unarmed trunk, stout branches forming a globose crown, and rather slender sparingly armed chestnut twigs, which are tomentose the first summer. Leaves ample, ovate or oblong-ovate, 7-12 cm. long, 6-8 cm. wide, rounded or cordate at base, rather coarsely serrate, 5-7 pairs of short broad notches, dark green, thin, soft pubescent below especially on the veins with short appressed pubescence; petiole 1-2 cm. long, tomentose. Inflorescence a many-flowered small compact tomentose corymb; flowers about 22 mm. in diameter, stamens 20. Fruit, borne in small com-

pact 5-8-fruited tomentose clusters, ripening about the middle of September and soon falling, is subglobose, 16-20 mm. thick, bright red, capped by the broad spreading, tomentose deeply serrate lobes.

Southern Illinois along streams but possibly extending into Missouri. *G. valens* is one of the early fruiting *molles*, and can be separated from *C. mollis* by the thinner more oblong leaves, not so tomentose below and less conspicuously cordate at base, and its smaller flowers.

CRATAEGUS VERNA. A tree becoming 7 m. in height with light brown furrowed bark on the unarmed trunk, and stout orange or light brown twigs, tomentose the first season and sparingly armed with slender 4-6 cm. long thorns. Leaves ovate or broadly ovate, 8-12 cm. long, rounded truncate or cordate at the broad base, with a few pairs of broad shallow notches, bluntly serrate, villose beneath with soft white pubescence, soon glabrate above; petiole slender, 3-6 cm. long, tomentose. Flowers, appearing the last of April or the first of May in large compound villose corymbs, are about 25 mm. wide; stamens large, 20. Fruit borne in compound few-fruited clusters on 2-3 cm.-long pedicels, ripening about the middle of September and soon falling, is subglobose, 16-18 mm. thick, dark red, and capped by the narrowly triangular serrate tomentose lobes.

River bottoms Eastern Missouri and Southern Illinois. Dr. N. M. Glatfelter, April 1895, September 1895; N. M. Glatfelter and W. W. Ashe, 1902. From *C. mollis*, Sch. and *C. valens*, above proposed, it is separated by its stout orange or russet twigs and villose corymbs.

CRATAEGUS VENOSA. A tree 6-8 m. in height with a broad oval crown, unarmed dark brown furrowed bark, and stout russet seldom armed twigs which are villose the first summer. Leaves broadly oval, 6-9 cm. long, truncate or cordate at the broad base, a few short spreading notches, sharply serrate nearly to the base, the 5-7 pairs of slender arched veins and the veinlets, deeply impressed, when young

tomentose below, at length glabrate or merely pubescent along the veins; petiole 2-4 cm. long, tomentose. The flowers, which appear in southern Illinois about the first week in May in compound many-flowered tomentose corymbs, are about 22 mm. wide; stamens small, 20. Fruit, borne in long-pedicelled tomentose clusters, ripening late in September and falling soon after, is globose or subglobose, 15-18 mm. thick, bright red, the narrow serrulate pubescent lobes often deciduous.

The broad, thin, acantily pubescent leaves, prominent thin arched primary veins, stout orange twigs and long-pedicelled fruit, separate this from the other late fruiting *molles* which have cordate leaves and globose fruit.

CRATAEGUS DISPESSA: *C. pyriformis* Brit., *Bul. N. Y. Bot. Gard.* I, 449(1900), not Rorn. *Fam. Nat. Syn.* III, 136 (1847).

CRATAEGUS NUPERA. A tree 5-6 m. in height with dark brown unarmed furrowed bark on the stout trunk, and stout russet glossy seldom armed geniculate twigs. Leaves ovate or broadly ovate, 5-11 cm. long, 4-10 cm. wide, acute or obtuse at apex, rounded or cordate at base, not lobed, or with a few pairs of very shallow notches near base, sharply serrate, tomentose beneath, as are the 2-5 cm.-long petioles. Flowers small, in small compact 6-10-flowered tomentose corymbs; stamens 20, small. Fruit, borne in small, 3-6-fruited, simple short-pedicelled tomentose clusters, on strict 1-2 cm.-long pedicels, ripening after the middle of October, and falling gradually, is globular or slightly oblong, 13-15 mm. long, dark red, capped by the lanceolate pubescent serrate reflexed lobes; flesh orange, sweet; seed usually 5, 7 mm. long.

Well separated from the other late-ripening species by the short compact corymbs, small globular fruit, and narrow reflexed calyx lobes. So far known only from Joliet, Illinois, where first collected by W. W. Ashe and J. H. Ferriss, October 8, 1901; and W. W. Ashe, 1902.

TENUIFOLIAE.

CRATAEGUS PRONA. A slender tree 4-7 m. in height with dark gray scaly bark on the sparingly armed trunk, and

rather slender sparingly armed glabrous chestnut twigs. Leaves glabrous, oval or broadly oval, 6-9 cm. long, 4-7 cm. wide, rounded or truncate at the base, acuminate at the apex, sharply and finely doubly serrate, several pairs of short broad notches with abruptly acuminate tips, bright green, thick; petioles slender 2.3 cm. long. The flowers, appearing the middle of May in somewhat compound glabrous corymbs, are about 20 mm. wide; stamens large, 5-8. The fruit, borne in nearly simple clusters on short drooping pedicels, ripening late in September and falling after the leaves, is oblong-obovate or pyriform, 12-19 mm. long, dark red mottled with green and russet, surmounted by the narrowly triangular entire lobes; flesh thin and firm; seed usually 3, 7-8 mm. long, deeply grooved on the broad round back.

Port Huron, Michigan, C. K. Dodge and W. W. Ashe; Mansfield, Ohio, W. W. Ashe.

CRATAEGUS FILIPES. A small bushy tree 7 m. in height with numerous slender ascending branches, and slender sparingly armed red-brown twigs. Leaves oval or broadly oval, 4-6 cm. long, acuminate at the apex, rounded or truncate at the base, with 3-5 pairs of deep acute spreading lobes, finely and sharply serrate, thick, dark green, glabrous; petiole 2-3 cm. long, very slender. The flowers, appearing the last week in May in large slightly compound glabrous corymbs, are 22 m. wide; stamens 5-8, anthers rose-purple. The fruit, borne in few-fruited compound clusters on very slender drooping pedicels, ripening late in October and falling after the leaves, is obovate, 12-14 mm. long, glossy crimson with bright green spots at the apex, sometimes sparingly pruinose, capped by the short broadly triangular entire spreading or ascending lobes, the tips of which are deciduous; seed usually 5, with a shallow furrow down the middle of the back.

Port Huron, Michigan; C. K. Dodge and W. W. Ashe. From *C. acutiloba* Sarg. it is separated by having larger flowers, firmer more glossy leaves, and dark crimson fruit mottled with green at the apex, which ripens several weeks later.

CRATAEGUS DECENS. A tree 5-7 m. in height with a short generally unarmed trunk, and numerous spreading sparingly armed branches forming an oval crown; bark on the trunk dark gray, broken by shallow furrows into low ridges, that on the branches lighter and smoother. Twigs flexuous, stout, glabrous, bright chestnut-brown, armed with short, 2-3 cm.-long, stout bright brown or gray curved thorns. Leaves oval or oblong-oval, 4-7 cm. long, 4-6 cm. wide, glabrous, thin, dark green above, paler beneath, rounded, subcordate or wedge-shaped at base, acuminate at apex, with 3-5 pairs of prominent veins running to the tips of the sharp-pointed lobes, sharply and coarsely serrate. The flowers, borne in 7-15-flowered nearly simple glabrous cymes, on 1-2 cm. long slender pedicels, are 21-25 mm. wide, calyx glabrous, obconic, the long triangular serrate lobes spreading after anthesis; stamens normally 20, anthers rose-pink. The fruit, borne in large generally simple clusters, is oblong, subglobose, or slightly obovate, 10-12 mm. thick, scarlet, often with orange spots at the apex, sometimes sparingly glaucous, and sometimes capped by the large green or brown spreading lobes with upcurved tips; cavity broad and shallow; flesh thick, orange-colored, soft and juicy; seed 3-5, usually 4, 5-6 mm. long, narrow across the slightly grooved back.

The flowers appear the last of May and the fruit ripens the middle of September and falls before and with the leaves early in October. Collected at Mansfield, Ohio, by E. Wilkinson 1901 and E. Wilkinson and W. W. Ashe, 1902, growing on sunny hillsides and in meadows. It is most closely related to *Crataegus lucorum* Sarg., from which its larger sharply serrate foliage, glabrous and not villose cymes, and mottled fruit separate it.

CRATAEGUS PARCA. A tree 5-8 m. in height with an oval crown of spreading branches, thin gray scaly bark on the unarmed trunk and slender sparingly armed red-brown twigs with large globose buds. Leaves ovate or broadly ovate, 6-8 cm. long, or rounded broadly cuneate at ths base, abruptly

acute at the apex, 3-5 pairs of short spreading lobes, sharply and finely doubly serrate, the lower pairs of slender primary veins arcuate; petiole slender, 2-4 cm. long. Inflorescence a many-flowered compound glabrous corymb; flowers about 18 mm. wide; stamens 5-8, small, The fruit, borne in small compound 4-8-fruited clusters, ripening early in October and falling with and after the leaves, is globose or slightly oblong, about 10 mm. thick, scarlet mottled with orange at the apex, capped by the very long broadly triangular entire glabrous reflexed lobes; flesh orange, sweet, juicy; seed 2-3, deeply sulcate on the broad rounded backs, or rarely 4 or 5, with narrow scarcely grooved backs.

Sunny woods, Milwaukee, Wisconsin. Very distinct from the other 5-8-stamened tenuifoliae, on account of its thin ample foliage, and coarse seed.

CRATAEGUS EXIGUA. A shrub with slender virgate stems 2-4 mm. in height, slender red-brown twigs freely armed with short 3-4 cm. long thorns, and dark gray scaly bark on the stems, the inner bark conspicuously orange. Leaves ovate, 4-7 cm. long, abruptly acuminate at the apex, 4 or 5 pairs of deep acuminate lobes, ascending with spreading tips, acuminately serrate nearly to the broadly cuneate base, thin, bright green, glabrous; the slender petiole 2-3 cm. long. The flowers, appearing about the middle of May in small many-flowered glabrous compound clusters, are about 16 mm. wide; stamens large, 5-8. The fruit, borne in small compound clusters, ripening the last of September and soon falling, is obovate, 12-14 mm. long, bright scarlet with orange spots at the apex, capped by the narrowly triangular entire spreading lobes, or lobes deciduous; cavity small, shallow, the tube not projecting; flesh soft and sweet; seed 3-5.

Milwaukee, Wisconsin. From *C. acutiloba* Sarg. and *C. Egani* Ashe, both small-fruited species, with 5-8 stamens, *C. exigua* can be separated by its larger foliage and nearly globose fruit.

CRATAEGUS PACTILIS. A tree 3-5 m. in height or a stout

bushy shrub with dark gray scaly bark and stout red-brown glabrous twigs sparingly armed with short 3-4 cm.-long thorns. Leaves ovate or oblong, ovate, 6-8 cm. long, gradually narrowed to the apex from the narrow rounded base, 4-6 pairs of short lobes with recurved tips, finely serrate nearly to the base, glabrous, thick, dark green, lucid; petiole slender, 2-3 cm. long. Inflorescence a small compound many-flowered glabrous corymb. stamens 5-8; calyx lobes narrowly triangular, serrulate. The fruit, borne in small compound clusters, ripening late in September, and falling largely before the leaves, is obovate, 13-16 mm. long, scarlet, orange or russet at apex, capped by the ascending bases of the lobes; flesh thick, sweet, orange; seed usually 5, 7-8 mm. long, attenuate at base.

Milwaukee, Wisconsin. Separated by its much larger, obovate fruit from the other 5-8-stamened species of the region.

PRUINOSAE.

CRATAEGUS ATER. A tree 4-6 m. in height, with dark gray finely furrowed bark on the thorny trunk and slender russet or red-brown glabrous twigs, often abundantly beset with 3-5 cm. long purple-brown thorns. Leaves, spreading on the slender 1-2 cm. long winged petioles, rhombic or broadly ovate in outline, 3-5 cm. long, somewhat narrower, finely serrate, with a few shallow ascending lobes above the middle, broadly cuneate or rounded at the entire base, glabrous, thick, dark green, turning dull yellow or brown and fallIng early in October. Flowers, appearing the last of May in slightly compound glabrous cymes, are about 20 mm. wide; stamens 20, anthers purple. Fruit, borne in simple few-fruited clusters, ripening late in October and falling after the leaves, is oblong or obovate, 11-13 mm. long, dark dull red mottled with green and olive, pruinose, capped by the narrow entire spreading lobes; tube prolonged; flesh greenish, hard; seed usually 3, large, thick and coarsely grooved.

Port Huron and St. Clair, Michigan, where collected by

C. K. Dodge and W. W. Ashe. From *C. latisepala* Ashe, with which it is associated, *C. ater* is separated by its differenly shaped foliage, oblong fruit and purplish anthers.

CRATAEGUS IMMANIS. A slender tree 4-7 m. high with rough black bark on the thorny trunk and very slender red-brown glabrous twigs freely armed with 3-4 cm.-long thorns. Leaves broadly ovate, 3-5 cm. long, 2-3 pairs of broad shallow notches, rather finely but bluntly doubly serrate, 2-4 pairs of primary veins, thick, firm, yellowish-green, glabrous; petiole short, 1-2 cm. long, winged, glandular. Inflorescence a nearly simple 4-15-flowered glabrous corymb; flowers about 20 mm. wide; calyx broad and shallow; stamens 20. The fruit, borne in simple few-fruited clusters on very slender 2-3 cm.-long pedicels, ripening early in October and falling after the leaves, is discoid, 13-16 mm. thick, 8-12 mm. long, bright green with large crimson spots; cavity very broad and shallow, the short broadly triangular serrate lobes reflexed; flesh greenish, hard; seed 5, usually with a single median groove down the narrow sharply angled back.

Dry hills, Port Huron, Michigan; C. K. Dodge and W. W. Ashe.

CRATAEGUS ONUSTA. A tree 5-8 m. in height with an oval crown of spreading branches, dark gray furrowed bark on the armed trunk, and slender red-brown glabrous twigs freely armed with dark 4-5 cm. long thorns. Leaves broadly oval, 4-7 cm. long, abruptly acute at apex, rounded at the broad base, sharply doubly serrate, with a few pairs of broad shallow irregular notches, thick, dark green, glabrous; petiole stout, 2-3 cm. long. The flowers, appearing the last week in May in 4-12-flowered nearly simple glabrous cymes, are about 20 mm. wide; stamens 20, anthers white. The fruit, borne in simple 3-10-fruited clusters on stout 2-2.5 cm.-long strict pedicels, ripening in October and falling gradually after the leaves, is globose, 14-16 mm. thick, dark red, glaucous, capped by the broad spreading or ascending entire lobes; tube broad, much prolonged; flesh hard, greenish; seed 4-5, a very

shallow groove on the flat back.

Ohio; E. Wilkinson and W. W. Ashe, Mansfield.

SILVICOLAE.

CRATAEGUS SECTILIS. A slender tree sometimes 7 mm. in height with a narrow oval crown, thin dark gray scaly bark on the armed trunk, and slender, nearly straight, red-brown glabrous twigs armed with many slender thorns. Leaves ovate, 3-4 cm. wide, 3.5-5 cm. long, taper-pointed, rounded at the base, rather thin but firm, glabrous, coarsely serrate, with a few pairs of very short broad notches, turning dull-brown and falling late in October; petiole slender, nearly round, 2-3 m. long. Inflorescence glabrous, small, 4-8-flowered, 4-5 cm. wide, somewhat compound, stamens 10, small. Fruit, borne in 3-6 fruited simple clusters, ripening about the middle of October and falling late in October or early in November with and after the leaves, is subglobose or obovate, 12-16 mm. thick, flattened at the ends, dark red, capped by the small entire persistent reflexed lobes which color with the fruit; flesh white, firm, sweet and juicy; seed central, usually 5; pedicels slender, seldom as long as the fruit.

Dry hills, McDowell and Mitchell Counties, N. C. The type from near Marion, N. C.

BOYNTONIANAE.

CRATAEGUS RESECTA. A small tree 5-6 m. high with short branches forming an oval crown, or a large irregular shrub, the bark on the trunk black and rough, the twigs glabrous, red-brown, armed with 4-5 cm.-long thorns. Leaves glabrous oval, 3-5 cm. wide, 4-6 cm. long, obtuse or rounded at the base, or sub-cordate on vigorous shoots, coarsely and bluntly serrate, with a few short broad notches; petiole about 2 cm. long, broadly winged above. Flowers about 20 mm. wide, borne in 6-10-flowered compound glabrous corymbs. Fruit globular, 14-17 mm. thick, flattened at the ends, slightly angled, bright scarlet, in 4-8-fruited often compound clusters; tube not projecting, the deeply glandular serrate ends of the

long lobes deciduous, the edges of the short spreading bases involute and coloring with the fruit; flesh thick, orange, juicy; seed usually 4, large and coarse, 6-8 mm. long.

Around Marion, N. C., *C. resecta* is one of the most common thorns, growing on poor land with oaks, hickories and pines. From *C. boyntoni*, Bead. of the same region it is separated by the more brightly colored fruit, deciduous lobes, with short involute bases, and the leaves not cordate even on vigorous shoots.

CRATAEGUS INDUCTA. A slender black-barked shrub, seldom exceeding 2 m. in height, with virgate stems and numerous short ascending branches. Twigs red-brown, glabrous, armed with numerous slender, 3-6 cm.-long thorns. Leaves glabrous, rather thin for the group, ovate, 3-4 cm. wide, 4-5 cm. long, acute or rounded at base, about three pairs of short sharp notches, finely serrate; petiole 1-2 cm. long, winged by the decurrent blade. Flowers, appearing the last of May in nearly simple 3-8-flowered cymes, are about 2-3 mm. wide, stamens large, normally 10, anthers pale pink; the very glandular bractlets larger and showy. The fruit, borne in nearly simple clusters on stout pedicels, is globose or obovate, dull green ar russet, ripens late in September and falls after the leaves; lobes broad, deeply serrate at apex, reflexed, persistent, tube projecting; seed usually three, slightly grooved on the rounded back; flesh greenish or white, hard.

Open woods and rocky pastures, eastern Pennsylvania, Kutztown; C. L. Gruber, May, 1902; C. L. Gruber and W. W. Ashe, September, 1902; Castle Rock, B. H. Smith and W. W. Ashe, September, 1902.

CRATAEGUS VIRGATA. A small tree or tall virgate shrub, 2-4 m. in height with short branches; the bark on trunk dark gray, the slender nearly straight twigs red-brown and sparingly armed with short 3-4 cm.-long thorns. Leaves ovate, 5-7 cm. long, 3-6 cm. wide, cuneate truncate or subcordate at base, glabrous, rather thin, yellowish-green, the distant curved veins prominently impressed, a few pairs of broad

shallow notches, sharply serrate; the slender petiole broadly winged above. Flowers, appearing the last week in May in nearly simple 4-7-flowered glabrous cymes, are about 24 mm. wide. Fruit, borne in simple 1-5-fruited clusters, ripening in October and falling gradually after the leaves, is depressed-globose, 12-16 mm. thick, dull russet red, sparingly mottled with green and orange, capped by the prominent projecting tube, occasionally the reflexed lobes persistent; flesh firm, greenish yellow; seed 3-4, 6-8 mm. long, deeply grooved on the rounded back; lobes triangular from a broad base, reflexed on the fruit.

Crataegus virgata is separated from the other white anthered species by its thin leaves with slender impressed veins, and the short triangular deciduous calyx-lobes. The type material was collected from hills along the South Fork of New River, near Elk Cross Roads, N. C.

CRATAEGUS LENTULA. A small tree exceptionally 6 m. in height with dark blackish rough armed bark on the trunk, short spreading branches, and slender glabrous red-brown sparingly armed twigs. Leaves ovate or broadly ovate, 2.5-4 cm. wide, 3-5.5 cm. long, cuneate to subcordate at base, sharply serrate with ascending teeth, several pairs of short broad obtuse lobes, yellow-green, thin but firm, erect or ascending on the stout 1.5-2 cm.-long margined glandular petiole. Inflorescence a small 3-6-flowered glabrous cyme; calyx lobes narrow, triangular-lanceolate, glandular serrate, reflexed. Fruit, borne in simple, few-fruited clusters, ripening and falling in October, with and after the leaves, is globular, 10-13 mm. thick, orange-red or dull-red; lobes reflexed, persistent; pedicels about the length of the fruit.

Open woods on hills along the Catawba River, near Catawba, N. C., where it is one of the most common thorns.

COCCINEAE.

CRATAEGUS LUMARIA. A tree 4-6 m. in height with dark nearly black rough thorny bark on the trunk, a flattened

crown of long horizontal branches, and stout glossy chestnut geniculate twigs freely beset with 3-6 cm.-long stout usually recurved thorns. Leaves obovate or orbicular, 3-6 cm. long, obtusely serrate, with broad shallow notches above the middle; rounded and broadly cuneate at the base, obtuse at the apex, glabrous, thick, lucid, erect and spreading on the 1½2 cm.-long petiole. Flowers, appearing the last week of May in small slightly compound pubescent corymbs, are about 15 mm. wide; stamens 5-10, small, anthers whitish. Fruit, borne in small 3-7-fruited nearly simple compact clusters, ripening late in October and falling after the leaves, is subglobose, 10-13 mm. thick, dull dark red, or mottled with green, capped by the narrow acute entire or serrate lobes, or the lobes deciduous; seed thick and coarse; flesh firm, hard, bitter and astringent.

From the other glabrous species of this group *C. lumaria* is separated by the large depressed, dull-colored, bitter fruit. C. K. Dodge and W. W. Ashe, Port Huron, Michigan.

CRATAEGUS DODGEI. A small bushy tree, 3-6 m. in height, with an open crown of few spreading branches and slender spreading dull brown geniculate twigs armed with numerous short, 3-5 cm. long, dark purple brown thorns. Leaves obovate or nearly orbicular, 2.5-4 cm. wide, or on vigorous shoots much larger, finely serrate, with several pairs of shallow lobes, cuneate at the entire base, dark green, thick, erect or ascending on the slender 1-2 cm.-long petiole, at first pubescent on the upper surface and petiole, at length nearly glabrous. Flowers, appearing at the type locality, Port Huron, Michigan, the last of May in small compound glabrous corymbs, are 16-18 mm. wide, stamens normally 10. Fruit, borne in 5-10-fruited small compact compound corymbs, ripening late in October and falling after the leaves, is depressed-globose, 12-20 mm. thick, often angled, dull crimson or orange, hard and bitter; seed mostly 3, large and coarse.

It affords me pleasure to associate the name of Mr. C. K. Dodge, the author of "The Flora of St. Clair County," with this tree.

Port Huron, Mich., where collected in 1901 and 1902 by C. K. Dodge and W. W. Ashe. Undoubtedly related to *Crataegus coccinea*, L., from which, and the other related northern species, separated by the large depressed dull-colored bitter fruit.

PUNCTATAE.

CRATAEGUS INDICENS. A slender tree 7-10 m. in height with black rough bark on the trunk armed with large compound thorns, short ascending branches, and slender upturned dull brown or russet twigs, at first villose, at length pubescent or glabrate, sparingly armed with 2-4 cm.-long ascending thorns. Leaves ovate or broadly ovate, 6-8 cm. long, rounded or broadly cuneate at the broad base, finely serrate from near the base, 3-5 pairs of short ascending lobes from below the middle, dark green above, brown pubescent beneath, at first tomentose; petiole slender, 3-4 cm. long, brown pubescent. Inflorescence a villose compound corymb; stamens 20, lobes pubescent serrate. The fruit borne in slightly compound few-flowered clusters on strict pubescent pedicels, ripening early in October and falling after the leaves, is globose, 13-15 mm. thick, dark dull red mottled with olive, pruinose, capped by the persistent reflexed lobes; flesh hard, white, firm; seed 3-4, shallow grooved on the back, 6-7 mm. long.

Rocky hills, Mansfield, Ohio: W. W. Ashe. The above very distinct species resembles *C. tomentosa* in habit, but in fruit and foliage it is evidently allied to the *punctatae*.

CRUS-GALLI.

CRATAEGUS TRAHAX. A tree 4-6 m. high with a broad flat-topped crown of slender horizontal branches, dark gray scaly bark on the armed trunk, and slender glabrous glaucous olive or dull brown twigs sparingly armed with 2-4 cm.-long thorns. Leaves, when mature, glabrous, thick, firm, dark green spatulate, oblong or oblong-obovate, 4.5-7 cm. long, bluntly doubly serrate at the rounded or acute apex, the long cuneate base entire, turning a dull yellow in October. The flowers, appearing the last week of May, in small 4-8 cm.-

wide many 12-18-flowered, compound glabrous corymbs, are about 16 mm. wide, cup-shaped and fragrant; stamens 8-10, anthers dark pink. The fruit borne in large loose compound clusters, and falling largely after the leaves, is globose, rarely slightly oblong, 8-10 mm. thick, crimson or dark red, mottled especially at the apex with green and olive, the narrow usually entire lobes spreading or ascending; flesh greenish, firm, bitter; seed 1-2, oblong, 6-8 mm. long with shallow grooves on the rounded back.

One of the most common thorns in Berks County, Pennsylvania; C. L. Gruber and W. W. Ashe.

CRATAEGUS IMMITIS. A tree 4-6 m. in height, with dark nearly black bark on the armed trunk, and slender glossy brown or chestnut glabrous twigs, freely armed with stout 4-6 cm.-long thorns. Leaves, small, thick, glabrous, dark green and lucid above, obovate or spatulate, 3-7 cm. long, rounded, mucronate or acute at the broad apex, cuneate or broader at the base, finely but sharply doubly serrate, often nearly to the base. Flowers, appearing about the first of June in small compact many-flowered compound glabrous corymbs, are about 14 mm. wide; stamens 8-12. Fruit, borne in small compound many-fruited clusters, ripening late in September and falling after the leaves, is globose, 9-12 mm. thick, dark red mottled with green at the apex, pruinose, capped by the very narrow, spreading, entire or distantly serrate lobes; flesh hard and greenish; seed 1-3, ovate, 5-7 mm. long, with shallow furrows on the round back.

Eastern Pennsylvania; W. W. Ashe, Delaware County. Separated from the other globose fruited species by the small doubly serrate foliage.

CRATAEGUS HELVINA. A slender tree 4-7 m. in height with nearly black scaly bark on the armed trunk, long, horizontal branches and slender russet or chestnut twigs armed with small 4-5 cm.-long thorns. Leaves thick, firm, glabrous, lucid above, oblong-ovate, spatulate or oblanceolate, 5-8 cm. long, acute or rounded at the apex, cuneate at the base finely

serrate nearly to the base on acute leaves, only above the middle on spatulate ones. Inflorescence a many-flowered loose compound glabrous corymb; flowers about 14 mm. wide, stamens 8-13. Fruit, borne in loose drooping many-fruited clusters, ripening late in October and falling after the leaves, is obovate or subglobose 10-14 mm. thick, light red, capped by the small usually entire reflexed or spreading lobes; flesh often pink; seed usually 2, seldom 1, 7-8 mm. long, oval with broad shallow furrows down the round back.

Common in Eastern Tennesssee on sandy soils. Type from near Clarkton. From *C. algens* Bead., which it resembles in its foliage, it can be separated by its larger obovate fruit.

CRATAEGUS PANDA. A bushy tree 3-6 m. in height with dark gray scaly bark on the armed trunk and slender brown glabrous twigs sparingly armed with slender 3-5 cm.-long thorns. Leaves small, 2-4 cm. long, obovate or spatulate, acute, rounded or mucronate at the apex, cuneate or broader at the entire base, finely serrate above the middle, or on vigorous shoots sharply doubly serrate and sparingly notched at the apex, dark green and shining above, thick, glabrous; petiole 3-8 mm. long, winged. Flowers, borne in loose compound glabrous corymbs, appearing the middle of May, are 12-14 mm. wide; stamens about 10. Fruit, borne in loose compound clusters, ripening in September, is oval, 12-14 mm. long, dull red, capped by the narrow entire spreading lobes; seed usually 2, 7-8 mm. long.

Sandy soil, Glendon, Moore County, North Carolina, growing beneath oaks and pine.

CRATAEGUS OPERTA. A tree 5-10 m. in height with a trunk seldom more than 2 dm. in thickness, armed with numerous simple or compound stout dark gray thorns, and with many long horizontal or spreading branches forming an oval or globose crown; the bark on the trunk dark gray or nearly black, broken into small scales; that on the branches smoother and light gray; twigs at first sparingly pubescent with short

spreading hairs, at length glabrate, stout, very tough, straight or flexuous, bright brown or brown-purple the first season, becoming dull gray the second year, armed with very numerous, stout, 4-8 cm.-long, curved gray or brown-purple thorns. Leaves dark green on both surfaces, though darker above, thick and firm, the broad veins not prominent, glabrous, or at first pubescent on the short petiole, obovate or broadly spatulate in outline (or on vigorous shoots broadly elliptic), 4-9 cm. long, sharply serrate at the rounded or acute apex, entire at the cuneate base and on the sides, turning dull yellow or brown in autumn and falling in October before the fruit; petiole short or the leaves nearly sessile. Inflorescence a many-flowered compound corymb, glabrate or more or less pubescent with short spreading hairs; flowers on slender nodding pedicels 12-15 mm. wide, cup-shaped, the large orbicular petals contracted at base into a short claw, erose denticulate at the apex; calyx obconic, glabrate, the narrow triangular lobes entire or serrulate, spreading after anthesis; stamens 8-14, large, purple. The fruit, borne in large glabrate clusters on drooping pedicels, is oblong, 12-15 mm. long, dark red when ripe in October, and capped by the persistent calyx-lobes; seed usually two, 7-8 mm. long.

The species above proposed is related to the cockspur thorn or perhaps more closely to *C. elliptica* Ait. From the first it is well separated by the pubescent inflorescence and twigs, the larger flowers with purple anthers and broader foliage, and from the second by the thicker usually obtuse leaves. It has been collected in both flower and fruit from the banks of the French Broad River in Madison County, North Carolina, and the Pigeon River in Cocke County, Tennessee.

CRATAEGUS ATTENUATA. A tree 4-6 m. in height with a globose crown of horizontal branches, dark gray nearly black rough bark on the armed trunk, and slender russet or chestnut twigs armed with 4-5 cm.-long thorns. Leaves obovate, oblanceolate, or on vigorous shoots oval, 3-6 cm. long, abruptly acute or rounded at the apex, cuneate at base or nar-

rowed into the 5-10 mm.-long petiole, finely doubly serrate above the middle, thick, glabrous, dark green. The flowers, borne in large loose many-flowered glabrous corymbs, appearing early in June, are about 14 mm. wide. Fruit, borne in large loose compound clusters, on drooping pedicels, ripening early in October and falling after the leaves, is oblong, 10-12 mm. long, dull red mottled with green or olive, capped by the subentire spreading lobes; flesh green and hard; seed usually 2, 7-8 mm. long, obovate, attenuate at base.

Southern Michigan and Northern Illinois. Type material collected at Port Huron, Michigan, by C. K. Dodge and W. W. Ashe. Well separated from the other related species with which associated, by its shorter and broader foliage and smaller fruit.

A METHOD OF STUDYING THE SEPTAL SEQUENCE IN PALEOZOIC CORALS.

BY J. E. DUERDEN.

In attempting to establish the morphological relationships of the various groups of coral and Actinians a greater significance attaches to the order of appearance and adult arrangement of the mesenteries and septa than to any other polypal or corallar characteristic. The adult symmetry is mainly radial, but development shows that this results from organs which arise bilaterally, not in cycles, according to a more or less definite succession characteristic of each group. In fossil corals the septal sequence can be definitely ascertained only where the narrow apex has been preserved. The usual method is to prepare sections of this region with a lapidary's wheel, and then grind to microscopic thinness. In most species, however, the septal plan is established within the first few millimeters from the apex, and it is rarely possible to secure more than one or two sections within such a limited region, and even then the correlation of one part with another may be uncertain. Hence, comparatively little has been established as to the development of the septa in Paleozoic corals, though this has by no means restrained speculation as to their relationships.

In a study of a series of Rugosa, from the Geological Department of the National Museum, the following method has been found fairly satisfactory for securing the details required.

About a centimeter is sawn off the narrow extremity of a coral having a perfect apex, and the cut surface is ground smooth and fixed by Canada balsam to a glass slide, as in the usual preparation of rock sections. The free tip is then ground with fine emery until the first traces of the primary septa are exposed. Under a low power of the microscope

the central line of calcification of the septa can be easily seen by reflected light as a lighter streak, especially if the surface be moistened. A camera lucida drawing of the primary septal plan is then made, after which the surface is again ground and examined from time to time until additional septa are displayed. Another drawing is taken, and the grinding proceeded with until a further stage is reached, when a corresponding figure is secured; and thus the process is continued so long as new septa are exposed, or until the septal succession is fully established.

The fragment being fixed to the slide the same orientation is retained throughout; hence there is no uncertainty in following each septum through the series, and inserting the new members as they make their appearance. As a rule, corals such as *Streptelasma* and *Zaphrentis* are completely calcified for some distance from the apex, without any interseptal spaces; but in all cases the median part of the septum stands out by reflected light as a distinct lighter line, corresponding with a darker line of calcification to be seen in sections viewed by transmitted light.

A series of figures is thus obtained which accurately represents the order of development of the septa for any individual coral, and it is manifest that the process can be continued without the omission of any stage. Where external ridges and grooves occur on the corallum they serve to confirm the results from sections, but the fundamental order is only to be ascertained at the earliest stages, where the ridges and grooves are generally obscure.

So far the primary septal plan of the simple Rugose corals has been found to be hexameral, not tetrameral, as hitherto has been generally assumed. By this the Paleozoic corals are brought into definite relationship with recent hexameral corals and Actinians. Beyond the six-rayed protoseptaltag se however the septal plan of the extinct corals diverges from living Madreporaria, proceeding in such a manner as to render it imperative to retain them as a distinct group, more allied to the living Zoanthids than to modern stony corals.

NOTE ON THE BROMINATION OF HEPTANE.

BY ALVIN S. WHEELER.

Venable [Amer. Chem. Jr. X. 237] found that the bromination of normal heptane from the *Pinus Sabiniana* yielded a mixture of bromides which varied in amount with the temperature and the time. Theoretical amounts of heptane and bromine were used but a large percentage of heptane was unattacked in every case. The secondary heptyl bromide was the main product when the heptane was kept boiling and the bromine was added during a course of five hours which was as rapidly as the bromine was taken up. The yield was thirty per cent. of the theory. This bromide is best obtained from heptane of the *Pinus sabiniana*, of California, which was formerly on the market under the name "abietine," a patent remedy. Small quantities only can now be obtained under great difficulties, as Blasdale [Jr. Amer. Chem. Soc. 23, 162] reports. The stock on hand in our laboratory is small and hence valuable. In view of this I thought it possible to increase the yield of the secondary bromide by carrying out the reaction under increased pressure. By means of air pressure the liquid bromine was forced into the flask containing the boiling heptane. A compact reflux condenser within the neck of the flask completely condensed the heptane. A side tube of the condenser led into a long glass jar, set horizontally, filled with stick potash to absorb the hydrobromic acid evolved. To the further end of the jar was fitted a tube, which, entering a jar of mercury vertically, opened at a point 190 mm. below the surface. Several attempts were made to increase the yield of the secondary bromide with this apparatus without success. I wish here to express my thanks to Mr. M. R. Glenn for his kind assistance in this work.

I then turned my attention to the possibility of accomplishing the result by employing an excess of bromine. The

method used by Venable was followed. Where he used 180 grams bromine for 100 grams heptane I used 240 grams bromine. The table shows the results after six complete fractionations, the first two being under diminished pressure.

		Theoretical Amount of Bromine	Excess of Bromine
I.	100° to 120°	30 grams	8 grams
II.	120° to 162°	5 "	2 "
III.	162° to 168°	55 "	65 "
IV.	168° to 173°	2 "	1.5 "
V.	173° to 183°	8 "	2 "
VI.	183° to 210°	10 "	45 "

I. consists principally of unattacked heptane and III. of the secondary heptyl bromide. The excess of bromine then considerably diminishes the amount of unchanged heptane; increases the secondary bromide by eighteen per cent., but causes a much greater increase in the bromides of higher boiling points. A further consideration of the table shows that 22 grams less of heptane are recovered, but as a compensation there is a gain of 10 grams of the secondary bromide. Now if 22 grams of heptane were treated with the theoretical amount of bromine only 6.6 grams of this bromide would be attained. Hence there is actually a net gain in the yield as well as a reasonably good gain in time. As the process is a long one these gains are worth consideration.